

TWO LOOP RENORMALISATION OF THE MAGNETIC COUPLING IN HOT QCD AND SPATIAL WILSON LOOP

P. GIOVANNANGELI

*Centre de Physique Théorique
163, avenue de Luminy,
13288, Marseille cedex 09, France
E-mail: giovanna@cpt.univ-mrs.fr*

With the known two loop renormalisation of the magnetic coupling, the 4D results of the spatial Wilson loop are compared to the prediction from the magnetostatic sector.

1. Effective theories for hot QCD

Perturbation theory in hot QCD suffer from infrared (IR) divergences. These divergences can be best confronted by constructing effective field theories for the low-energy dynamics. For the partition function and other static observables, the relevant theory is the Dimensionally Reduced (DR) effective theory^{1,2}.

The main idea of DR is to separate systematically the physical scales of the quark-gluon plasma and to construct, for each of these scales, an effective action. The procedure is as follows:

- At distances of order $1/T$, physics is described by the non-static modes. But, if one wants to go to larger distances, one meets with IR divergences, due to massless static modes.
- At distances of order $1/gT$, however, electrostatic modes get screened by the Debye mechanism, like in a classical electromagnetic plasma. This thermal mass is going to regulate the electric IR divergences. At these scale, QCD is described by a 3D electrostatic Lagrangian L_E that reads:

$$L_E = Tr(\vec{D}(A)A_0)^2 + m_E^2 Tr A_0^2 + \lambda_E (Tr(A_0^2))^2 + \\ + \bar{\lambda}_E (Tr(A_0)^4 - \frac{1}{2}(Tr A_0^2)^2) + \frac{1}{2} Tr F_{ij}^2 + \delta L_E. \quad (1)$$

with as parameters $g_E(g, T)$, $m_E(g, T)$, ... and δL_E represents higher orders operators of relative order g^4 .

- Furthermore, at distances of order $1/g^2 T$, magnetostatic modes are the dominant ones. At these distances QCD is described by a confining 3D Yang-Mills theory L_M that is non-perturbative.

$$L_M = \frac{1}{2} Tr F_{ij}^2 + \delta L_M. \quad (2)$$

with a gauge coupling $g_M(g_E, m_E, \dots)$. δL_M is of relative order g^3 .

If one wants to compute the thermal average of an observable, then each of these three scales will have its own contribution to the final result.

From the relative order of the truncation in (1) and (2) we see that g_M is needed to relative order g^2 . In the next section, I will briefly present the result for the coupling g_M of the 3D magnetostatic Yang-Mills theory L_M at two loop order.

2. Two-loop determination of the magnetostatic action

The basic idea behind the effective actions eqns (1) and (2) is that one can compute with both in the region of momenta $p \sim g^2 T$. To know what the parameters of the latter are in terms of those of the former requires computing two-point functions, three point functions etc. in both theories and match them. In the matching the diagrams of the pure 3d Yang-Mills theory drop out.

Here we will follow a well-known shortcut ³ by introducing a background field B_i in L_E :

$$\begin{aligned} \vec{A} &= \frac{1}{g_E} \vec{B} + \vec{Q} \\ A_0 &= g_E Q_0. \end{aligned} \quad (3)$$

We calculate the fluctuations around the background in a path integral:

$$\exp - \frac{1}{g_M^2} S_M(B) = \int DQ_0 DQ_i \exp \left(- S_E - \frac{1}{\xi} Tr(D_i Q_i)^2 \right). \quad (4)$$

We added a general background gauge term. The resulting action $S_M(B)$ is gauge invariant to all loop orders and the renormalization of the coupling is identified from the background field two point function at a momentum $p = O(g^2 T)$.

$$\exp\left(-\frac{1}{g_M^2}S_M(B)\right) = \exp\left(-\frac{1}{g_E^2}S_M(B)\right)\left(1+(F_1^{tr}+F_2^{tr}+\dots)S_M(B)\right). \quad (5)$$

F_i is the sum of all Feynman diagrams for the two point function of the background field with i loops and reads:

$$F_i = F_i^{tr} (\delta_{lm}p^2 - p_l p_m). \quad (6)$$

This leaves us with the relation, using eq.(5):

$$\frac{1}{g_M^2} = \frac{1}{g_E^2} - F_1^{tr} - F_2^{tr}, \quad (7)$$

with

$$g_E^2 F_1^{tr} = -\frac{1}{48} \frac{g_E^2 N}{\pi m_E} \quad (8)$$

$$g_E^2 F_2^{tr} = -\frac{19}{4608} \left(\frac{g_E^2 N}{\pi m_E}\right)^2. \quad (9)$$

This is the main result ⁴.

3. Spatial Wilson Loop

We want now to test the applicability of 3D physics at medium high T. Let us consider an observable which has, unlike the pressure whose dominant contribution is the Stefan-Boltzmann term due to hard modes, its dominant contribution from the 3D Yang-Mills sector. Such an observable can be the spatial Wilson loop in the fundamental representation:

$$W(L) = \text{Tr} P \exp\left(i \oint_L g \vec{A} \cdot d\vec{l}\right). \quad (10)$$

As L is purely spatial, it measures the magnetic flux in the plasma. The case where \vec{A} is in an irreducible representation made of k quarks is described in this volume by C.P. Korthals Altes. The thermal average of this spatial Wilson loop shows area behaviour with a surface tension $\sigma(T)$. As it is a purely magnetic quantity, we expect from dimensional arguments that $\sqrt{\sigma} = c g_M^2$ where c is a nonperturbative proportionality constant. Indeed, as the average of the loop is due to long distance correlation, hard modes will not have any effects on the thermal average. In the same way, for soft modes, we can integrate out the A_0 field, which is what we have done in

the previous section while constructing L_M , as the loop does not depend on A_0 ! Finally, we have:

$$\langle W(L) \rangle = \exp -\sigma A(L) = \frac{\int D\vec{A} W(L) \exp -S_M(A)}{\int D\vec{A} \exp -S_M(A)}, \quad (11)$$

and it gives, as δL_M is of relative order g^3 ,

$$\sigma(T) = c^2 g_M^4 (1 + O(g^3)). \quad (12)$$

Now the aim is to fit with our formula for g_M the proportionality constant c , and to study its extension for finite T . For that, we have to go to the lattice. We are going to fit $\frac{T}{\sqrt{\sigma}}$ has a function of $\frac{T}{T_c}$. Indeed, g_M is a function of g_E and m_E which are functions of g and T . So, for $N = 3$:

$$g_M^{-2} = g^{-2} T^{-1} \left(1 + \frac{g}{16\pi} + \frac{19g^2}{512\pi^2} \right) \quad (13)$$

with

$$g^{-2} = \frac{11}{8\pi^2} \left(\text{Log}\left(\frac{T}{T_c}\right) + \text{Log}\left(\frac{T_c}{\Lambda_{\overline{MS}}}\right) + 1.90835\dots \right). \quad (14)$$

So thanks to these formulas, one can fit $\frac{T}{\sqrt{\sigma}}$ versus $\frac{T}{T_c}$ in order to determine c and $\frac{T_c}{\Lambda_{\overline{MS}}}$.

We have taken data points for the $SU(3)$ spatial string tension from ^{6,9} for $T > 2T_c$. So we have 10 points. The fit is shown in fig 1.

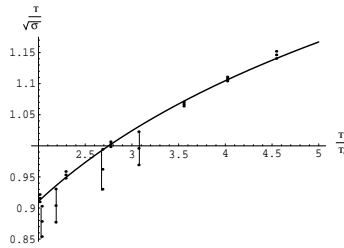


Figure 1. Plot of $\frac{T}{\sqrt{\sigma}}$ versus $\frac{T}{T_c}$ and fit with our two-loop formula. Each of the 10 points is shown with its error bar.

We have found the following results:

$$\frac{T_c}{\Lambda_{\overline{MS}}} = 1.78(12) \quad (15)$$

$$c = 0.505(11). \quad (16)$$

Then we have also fitted data with $T > 2.5T_c$. This leaves us with 6 points. We obtained

$$\frac{T_c}{\Lambda_{\overline{MS}}} = 1.57(20) \quad (17)$$

$$c = 0.488(18) \quad (18)$$

Let us draw our conclusions.

Despite the fact that the region of confidence is larger, the value of c is still incompatible with refs. ^{8,7,5}. To see compatibility one obviously needs string data for higher T . It might also be recommendable to use other Monte Carlo updates, like the one described in ref. ¹¹, which is a version of the Lüscher-Weisz algorithm ¹⁰.

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